APPENDIX D: THEORETICAL ANALYSIS OF UWB SIGNALS USING BINARY PULSE-MODULATION AND FIXED TIME-BASE DITHER

The theoretically derived spectra for fixed time-base dithered and binary pulse-modulated UWB signals were used to validate some of the test procedures described in this report. The calculated power spectral densities and examples showing how the analytical results compare to measurements are given below. The analytical results presented in this section are taken from [1].

Fixed time-base dithered UWB systems utilize short duration pulses transmitted at some nominal pulse period *T*. In this scheme, pulses are dithered about integer multiples of *T*. In the following discussion, it is assumed that the dither times are random variables θ_n that are independent and identically distributed over a fraction of the nominal pulse period with probability density $q(\theta_n)$. UWB signals may also include information bits by using binary pulse modulation in addition to the pulse dithering.

The power spectral density for a binary pulse-modulated fixed time-base dithered UWB signal is obtained by taking the Fourier transform of the autocorrelation function. Due to the periodic nature of the underlying pulsed signal, the process is *cyclostationary* with period *T*. Time averaging the autocorrelation function over a period yields the average power spectrum which depends only on the relative time delay. The time averaged power spectral density for a fixed time-base dithered UWB signal with binary pulse modulation is

$$
\begin{aligned}
\bar{R}_{xx}(f) &= L + C \\
L &= \frac{1}{T^2}\left|\sum_{k=0}^{1} g_k P_k(f)\right|^2 |Q(f)|^2 \sum_n \delta(f - n/T) \\
C &= \frac{1}{T}\left[\sum_{k=0}^{1} g_k |P_k(f)|^2 - \left|\sum_{k=0}^{1} g_k P_k(f)\right|^2 |Q(f)|^2\right]
\end{aligned}
\quad, \qquad \text{D.1}
$$

where P_k is the Fourier transform of the signal pulse for the information bit having the value *k*, *Q* is the Fourier transform of probability density function that describes the dithering, and g_k is the probability that an information bit has the value *k* (e.g., g_o is the probability that an information bit is “0” and g_1 is the probability that the bit is a “1”). Note that *L* is discrete (i.e., spectral lines) and *C* is continuous.

D.1 Fixed Time-base Dither and Pulse Position Modulation

If the bit values are equiprobable (i.e., g_k = ½) and the pulse representing a 1 is a time delayed version of the pulse representing a 0 (i.e., $p_1(t+\tau) = p_0(t) = p(t)$), the power spectral density becomes

$$\begin{aligned} \bar{R}_{xx}(f) &= L + C \\ L &= \frac{1}{2T^2} \left|P(f)Q(f)\right|^2 \left[1+\cos(2\pi\tau f)\right] \sum_n \delta(f-n/T) \\ C &= \frac{1}{T} \left|P(f)\right|^2 \left(1 - \frac{\left|Q(f)\right|^2\left[1+\cos(2\pi\tau f)\right]}{2} \right) \end{aligned} \qquad , \qquad \text{D.2}$$

Note that the discrete and continuous components depend on both the pulse spectrum and $Q(f)$. When $Q(f) \rightarrow 1$ (negligible dithering) and the information bits do not change, the continuous spectrum disappears leaving only a line spectrum as would be expected for a simple periodic pulsed signal.

The results of an example calculation using Equation D.2 when q is uniformly and continuously distributed between 0 and $T/2$ is given below. For this example, the signal consists of a short-duration pulse, shown in Figure D.1.1, transmitted at a 20 MHz rate. In this and following examples, it is assumed that τ is small in comparison to the dithering, so that the effects of information bit modulation are negligible over the frequency range of interest.

The power spectral density over a frequency range of 1-5000 MHz is shown in Figure D.1.2. The magnitude of the spectrum is normalized to the peak of the continuous distribution (at about 250 MHz). The Fourier transform of the density function for this example is $Q(f) = \text{sinc}(\pi f T/2)$. This function has nulls at frequencies equal to $2k/T$ ($k = \pm1, \pm2, \pm3, ...$); hence the interval between discrete spectral lines is 40 MHz, as shown in the figures. For frequencies above about 40 MHz, the continuous spectrum is approximately the same as the pulse spectrum (i.e., $P(f)$).

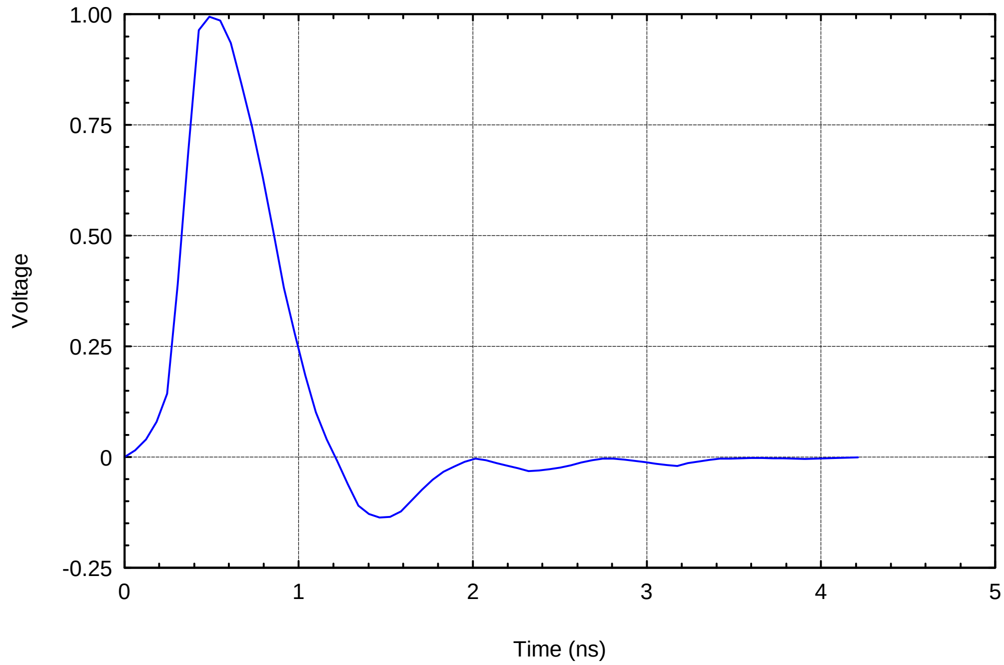

Figure D.1.1. Time-domain pulse shape.

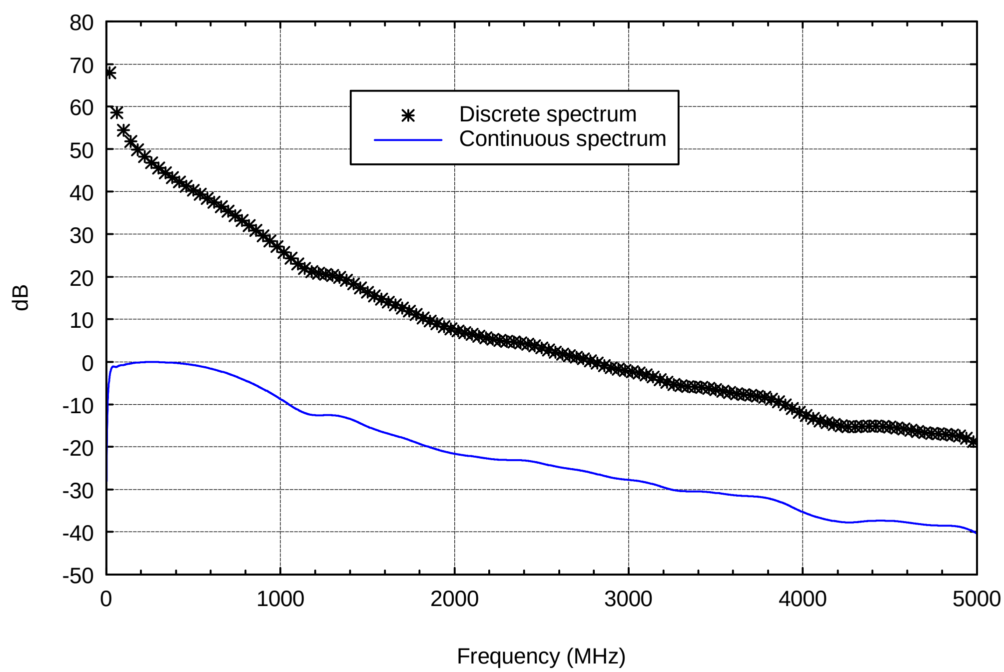

Figure D.1.2. Power spectral density for a fixed time-base dithered 10-MHz UWB signal. The pulse positions are continuously and uniformly distributed over 50% of the pulse repetition period.

The mean power in the bandwidth of a *narrowband* victim RF receiver as a function of frequency can readily be calculated from these results. For narrowband receivers where gains due to the UWB transmitter filters/antenna, propagation channel, and receiver are approximately constant over the receiver bandwidth, the received interference power can be calculated by applying the appropriate gain factors to the power in the receiver bandwidth at the center frequency of the receiver.

In the previous example, q is continuous and uniformly distributed over a fraction of the nominal period T. When the distribution is discrete so that the dithered pulse can only occur at particular times (e.g., $T - n\tau$, where $n = 0, 1, 2, 3, \cdots, N - 1$) with equal probability, the density function can be written as

$$q(t) = \frac{1}{N} \sum_{n=0}^{N-1} \delta(t - n\tau) \quad , \qquad \text{D.3}$$

with spectrum

$$|Q(t)|^2 = \left(\sum_{n=-\infty}^{\infty} \operatorname{sinc}\left[\pi N \tau (f - n/\tau)\right] \right)^2 \quad , \qquad \text{D.4}$$

which is a periodic function with period $1/\tau$. For example, when $1/T$ = 20 MHz, and the pulse is discretely dithered over one half of the pulse-repetition interval with τ = 1 ns, the spectrum is repeated at 1-GHz intervals as shown in Figure D.1.3. In this example, the receiver bandwidth is 1 MHz and the continuous spectrum is normalized to a maximum of 0 dB.

Note that for any integer m

$$|Q(m/\tau)|^2 = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \quad ,$$

hence, the continuous spectrum decreases to a minimum at integer multiples of 1 GHz. For these frequencies, the discrete spectrum tends to a local maximum and spectral lines are significant. Contrast this with the case where q is continuous (i.e., $Q(f) = \operatorname{sinc}(\pi f T/2)$) described previously. With continuous dithering, spectral lines at multiples of 1 GHz are not present, since they occur at nulls of $\operatorname{sinc}(\pi f T/2)$.

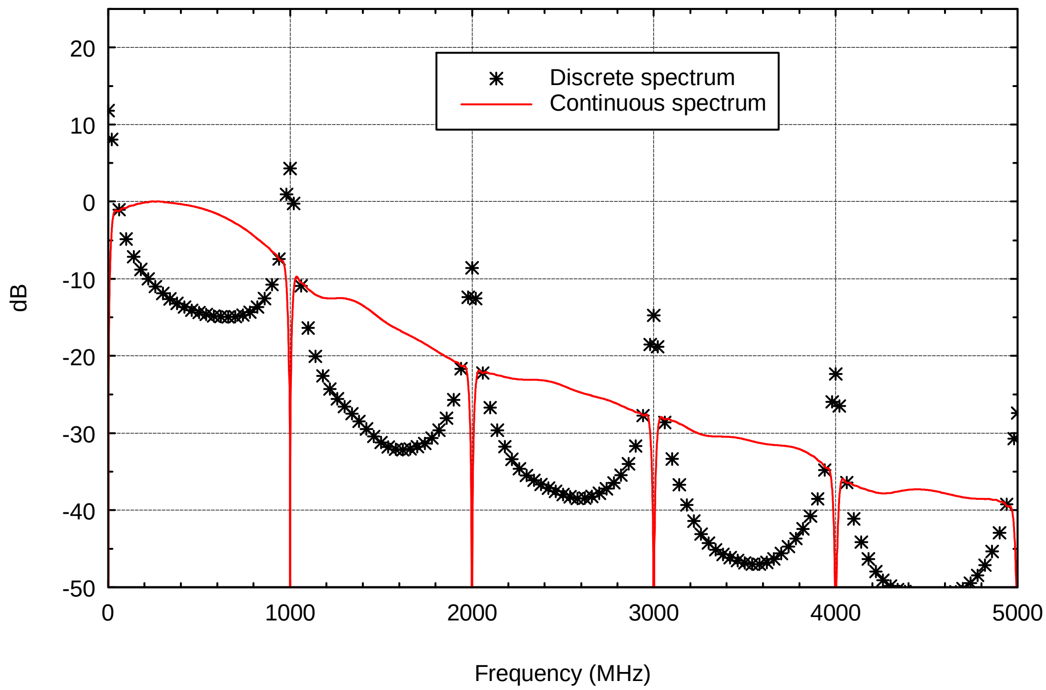

Figure D.1.3. Power spectral density for 20-MHz PRF 50% uniform discrete dithering with $J = 1$ ns.

A comparison of measured and predicted spectra for a discretely dithered UWB signal is shown in Figure D.1.4. The UWB signal is pulsed at a 20 MHz rate with uniform 50% discrete dithering with $J =$ 1 ns. The measurement bandwidth is 1 MHz. As predicted, only three lines, the strongest at 1 GHz and two others at 1 GHz ± 20 MHz are visible in the measured signal. This figure shows good agreement between measurement and theory.

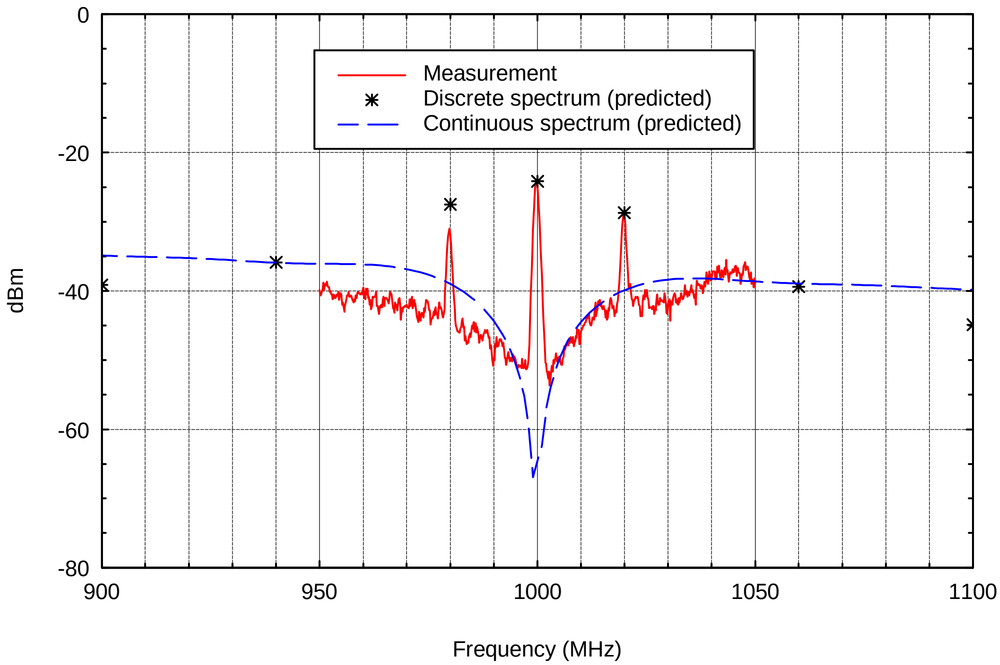

Figure D.1.4. Comparison of measured and predicted spectra for 20-MHz PRF 50% uniform discrete dithering with J = 1 ns.

D.2 Power Spectrum for On-off Keying Without Dithering

For binary pulse modulation using on-off keying without dithering, we set $P_0 = P(f)$, $P_1 = 0$, $Q(f) = 1$ and $g_0 = g_1 = ½$ in Equation D.1 and obtain

$$\begin{aligned} \bar{R}_{xx}(f) &= L + C \\ L &= \frac{|P(f)|^2}{4T^2} \sum_n \delta(f - n/T) \\ C &= \frac{|P(f)|^2}{4T} \end{aligned} \quad ,$$

When the signal is passed through a narrowband receiver with center frequency f_c and the bandwidth B, the received power is

$$\int_{f_c - B/2}^{f_c + B/2} \bar{R}_{xx}(f)\, df = \frac{|P(f_c)|^2}{4T^2}\left[N + TB\right] \quad ,$$

where N is the nominal number of lines in the filter passband. The ratio of the power in bandwidth B due to discrete and continuous components of the signal is simply $N(TB)^{-1}$.

Figure D.2.1 shows the spectrum of a signal generated by test equipment using on-off keying with equiprobable random bits and a pulse repetition frequency of 1 MHz. The signal was passed through a 20 MHz bandpass filter and a spectrum analyzer using a resolution bandwidth of B = 100 kHz. In this case N = 1, and hence, $(TB)^{-1}$ = 10 which is in agreement with Figure D.2.1 where the discrete spectrum is roughly 10 dB above the level of the continuous spectrum.

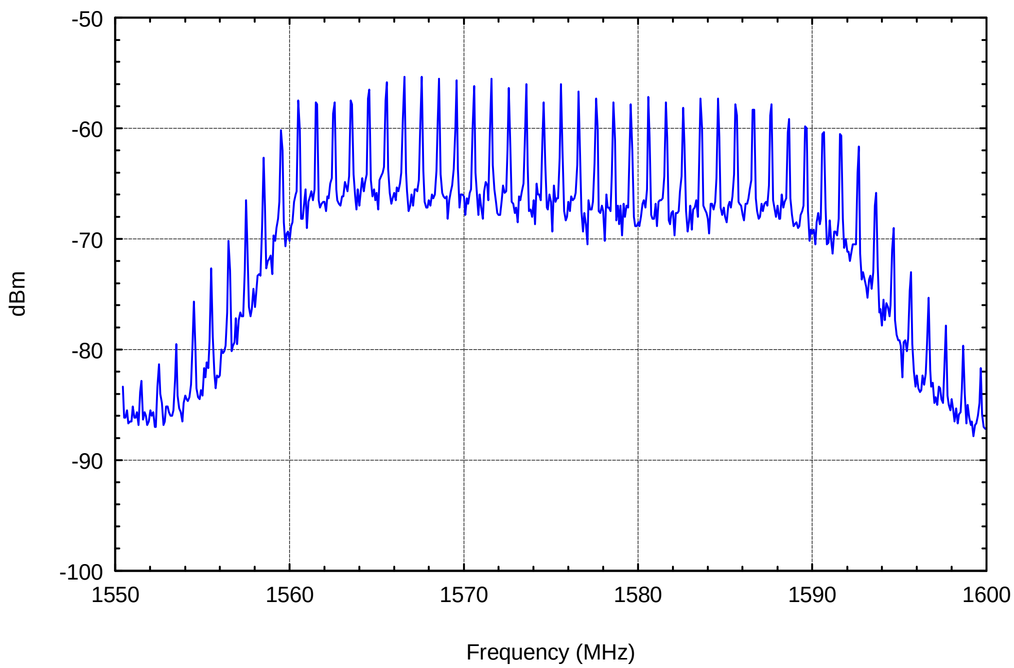

Figure D.2.1. Measured spectrum for on-off keying at 1-MHz PRF, B = 100 kHz.

References

[1] W.A. Kissick, Ed., “The temporal and spectral characteristics of ultrawideband signals,” NTIA Report 01-383, Jan. 2001.

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